

5. Exercise

Dependable Systems

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Task 1

Theoretical Questions:

- a) What fundamental difference lies between the Markov Chains/Petri Nets and the previous combinatorial approaches?
- b) What is the main motivation to use more complex models for reliability analysis?
- c) What is the main property of the Markov Chains? Where are they useful?
- d) What can be said about the probabilities of the state changes? How are they constrained?
- e) What is the main drawback of a Markov Chain?
- f) What is the difference between DTMC and CTMC?
- g) What kind of systems can be described by Petri nets?
- h) Define a Petri net formally.

Task 2

Consider an example where the probability that a sunny day is followed by a sunny day is 90% and a rainy day will be 50% likely to be followed by another rainy day. What are meaningful probabilities that a sunny day will be followed by a rainy day and vice versa? Make a graph with the Markov chain. Create a transition matrix, compute the probabilities for weather for 2 days time, starting with a sunny day. What is the steady state of the weather?

Task 3

Consider the Reader-Writer-Problem and model it with a classical Petri net. Construct a reachability graph for the system and prove that mutual exclusion is indeed enforced.

Task 4 (optional)

It is your task to analyze the availability behavior of a tiny data center environment. It consists of 45 servers that run in a redundant setup, providing services for the customer. 5 additional servers are used as cold standby backup. Due to power supply and cooling restrictions in the tiny data center building, no more than 45 servers should run at the same time. If less than 45 run, the data center is assumed to be broken, since performance level agreements are violated.

There are two administrators capable of performing repair activities, Jens and Ralf. Both can only repair one machine at a time. Jens needs on average 48 hours for one machine, while Ralf needs 96 hours on average - he is a busy man. Bringing a cold spare machine to life is only possible by an explicit activation step that takes 1 hour on average. Jens and Ralf can do this in parallel to their normal work, since the complete process works in an automated fashion (software image deployment, unattended install support). Both give their best to operate the center always at maximum performance. If there is something to repair, they do not sleep or eat.

Since all servers were bought from, Dall' company, they have the same failure rate. On average, one machine breaks per week. All stochastic processes in this exercise follow the exponential distribution. Assume 45 machines as running and 5 machines in the spare pool in the start configuration.

- a) What is the availability of the data center ? Explain the formula in relation to your Petri net.
- b) If each working machine can handle 1000 requests per hour on average, what is the average throughput of the system ?

Source: Peter Tröger - Dependable Systems, Uni Potsdam 2013

Literatur

- [1] Kai Lai Chung - Elementary Probability Theory, 2006
- [2] Tadao Murata - Petri Nets: Properties, Analysis and Applications, 1989 IEEE Invited Paper
- [3] J.R. Norris - Markov Chains, 1998 Cambridge University Press